**Part 3: Urban Population Dynamics**

A =

0 1.2 1.1 .9 .1 0 0 0 0

.7 0 0 0 0 0 0 0 0

0 .85 0 0 0 0 0 0 0

0 0 .9 0 0 0 0 0 0

0 0 0 .9 0 0 0 0 0

0 0 0 0 .88 0 0 0 0

0 0 0 0 0 .8 0 0 0

0 0 0 0 0 0 .77 0 0

0 0 0 0 0 0 0 .40 0

1. A Leslie matrix is a square matrix that has the same dimensions as the length of an input population vector. Each cell  (i, j) corresponds to the amount of individuals that will be in age class i for each individual in stage j. In constructing a Leslie matrix, it is necessary to know the count of individuals in each age class x, the fraction of surviving individuals from class x to class x+1 and the fecundity of each class x. The matrix’s top row is comprised of the fecundities, while the submatrix formed by ignoring the top row and final column is full of zeroes except on the pivot positions, which are the survival rates for each age class. The model that uses the Leslie matrix suggests a stable growth rate and age-structure. The population tends asymptotically to this structure and rate regardless of initial population size.

2. The distributions are as follows:

Current year: 2010

Current Population: 1.7349E7

6350000.000000 1470000.000000 1615000.000000 1620000.000000 1890000.000000 1760000.000000 1360000.000000 924000.000000 360000.000000

Percent change: 22.176056338028168

Current year: 2020

Current Population: 1.82815E7

5187500.000000 4445000.000000 1249500.000000 1453500.000000 1458000.000000 1663200.000000 1408000.000000 1047200.000000 369600.000000

Percent change: 28.742957746478876

Current year: 2030

Current Population: 2.212124E7

8162400.000000 3631250.000000 3778250.000000 1124550.000000 1308150.000000 1283040.000000 1330560.000000 1084160.000000 418880.000000

Percent change: 55.78338028169014

Current year: 2040

Current Population: 2.65050467E7

9656485.000000 5713680.000000 3086562.500000 3400425.000000 1012095.000000 1151172.000000 1026432.000000 1024531.200000 433664.000000

Percent change: 86.65525845070422

Current year: 2050

Current Population: 3.387942932E7

13413226.750000 6759539.500000 4856628.000000 2777906.250000 3060382.500000 890643.600000 920937.600000 790352.640000 409812.480000

Percent change: 138.58753042253522

3. The dominant eigenvalue was 1.2886561990635659. The dominant eigenvalue is the growth rate of the population. A growth rate over 1 indicates the population will eventually stable out. Lower than 1 indicates that the population will eventually die out. If the power method never converges then the population is unstable in the long run.

4. The output for this procedure is as follows:

Current year: 2030

Current Population: 1.6209E7

5210000.000000 1470000.000000 1615000.000000 1620000.000000 1890000.000000 1760000.000000 1360000.000000 924000.000000 360000.000000

Percent change: 14.147887323943662

Current year: 2040

Current Population: 1.66015E7

4305500.000000 3647000.000000 1249500.000000 1453500.000000 1458000.000000 1663200.000000 1408000.000000 1047200.000000 369600.000000

Percent change: 16.911971830985916

Current year: 2050

Current Population: 1.767974E7

5016600.000000 3013850.000000 3099950.000000 1124550.000000 1308150.000000 1283040.000000 1330560.000000 1084160.000000 418880.000000

Percent change: 24.505211267605635

Converged after 35 iterations.

Dominant eigenvalue: 1.167902832734512

Dominant eigenvector:

1.000000

0.599365

0.436218

0.336155

0.259045

0.195187

0.133701

0.088149

0.030191

This eigenvalue again indicates that the population has a positive growth rate that will eventually be stable. This rate is less than the previous one, so growth will be slower in this case.